

# 

### Semester Two Examination, 2019

### Question/Answer booklet

# MATHEMATICS

If required by your examination administrator, please place your student identification label in this box

**METHODS**

**UNITS 3 AND 4**

## Section Two:

## Calculator-assumed

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working  time (minutes) | Marks available | Percentage of examination |
| Section One:  Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two:  Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
|  | | |  | **Total** | 100 |

|  |  |  |
| --- | --- | --- |
| Markers use only | | |
| Question | Maximum | Mark |
| 9 | 6 |  |
| 10 | 6 |  |
| 11 | 6 |  |
| 12 | 6 |  |
| 13 | 9 |  |
| 14 | 8 |  |
| 15 | 8 |  |
| 16 | 8 |  |
| 17 | 7 |  |
| 18 | 7 |  |
| 19 | 9 |  |
| 20 | 9 |  |
| 21 | 9 |  |
| S2 Total | 98 |  |
| S2 Wt (×0.6633) | 65% |  |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.  
Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (98 Marks)

This section has**thirteen (****13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 (6 marks)

The graph of , where , has a root at .

(a) Determine the value of the constant and hence state the equation of the asymptote of the graph. (2 marks)

(b) Determine the exact coordinates of the point on the graph where . (3 marks)

(c) The graph of is congruent with the graph of . State a suitable function . (1 mark)

Question 10 (6 marks)

A machine is set to fill bottles with more than the stated capacity. The random variable mL is the amount it overfills bottles and has probability density function shown below.

(a) Determine . (2 marks)

(b) Determine . (2 marks)

(c) The amount another machine overfills bottles is given by . Determine

(i) . (1 mark)

(ii) . (1 mark)

Question 11 (6 marks)

A water tank sprung a leak. The amount of water remaining in the tank minutes after the leak began can be modelled by the equation kilolitres, where is a constant.

kL of water was lost from the tank in the first minutes.

(a) Determine the value of . (2 marks)

(b) How many kilolitres of water leaked from the tank during the first hours? (2 marks)

(c) At what time, to the nearest minute, was the instantaneous rate of water loss 186 litres per minute? (2 marks)

Question 12 (6 marks)

An opinion poll found that out of people supported a policy to increase the minimum wage, from which a approximate confidence interval for the population proportion was calculated to be

(a) Show how this interval was calculated. (4 marks)

(b) If similar opinion polls were taken and each time a confidence interval calculated, determine the probability that all intervals contain the true population proportion.

(2 marks)

Question 13 (9 marks)

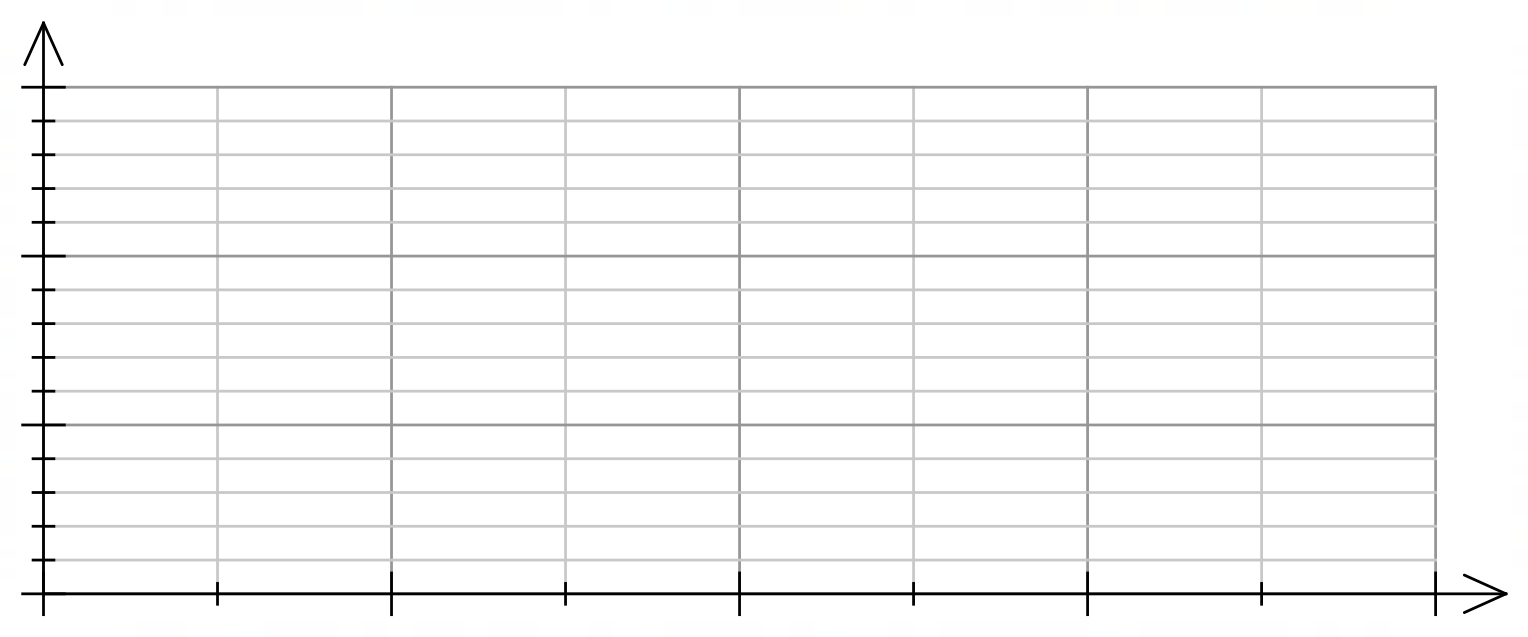
The time taken to answer a customer call at a large business can be modelled by the continuous random variable that is uniformly distributed between and seconds.

(a) Sketch a diagram of the associated probability density function for . (3 marks)

(b) Determine . (2 marks)

(c) A simulation involves taking a random sample from the uniform distribution, recording the time and repeating a total of times. The times are then grouped into equal width classes, from which a frequency histogram is constructed.

(i) Sketch a likely histogram on the axes below. (3 marks)



(ii) Briefly explain how your sketch would change if the simulation was repeated a second time. (1 mark)

Question 14 (8 marks)

The table below shows the probability distribution for a random variable .

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  |  |  |  |  |

(a) Determine the value of the constant . (2 marks)

(b) Determine and . (3 marks)

(c) Given that and , determine all possible values of the constants and . (3 marks)

Question 15 (8 marks)

It is known that of a large population of animals carry microfilariae in their blood (are carriers). A student must simulate selecting animals that either are or are not carriers.

(a) Describe a method that the student could use. (2 marks)

(b) The random variable is the number of animals in a random sample of size that are carriers. Describe the distribution of and determine . (2 marks)

students carry out the simulation so that they each have a sample of size . Then each student calculates , the proportion of animals in their sample that are carriers. The distribution of these values of will be approximately normal.

(c) Determine the parameters of the normal distribution the values of will approximate. (2 marks)

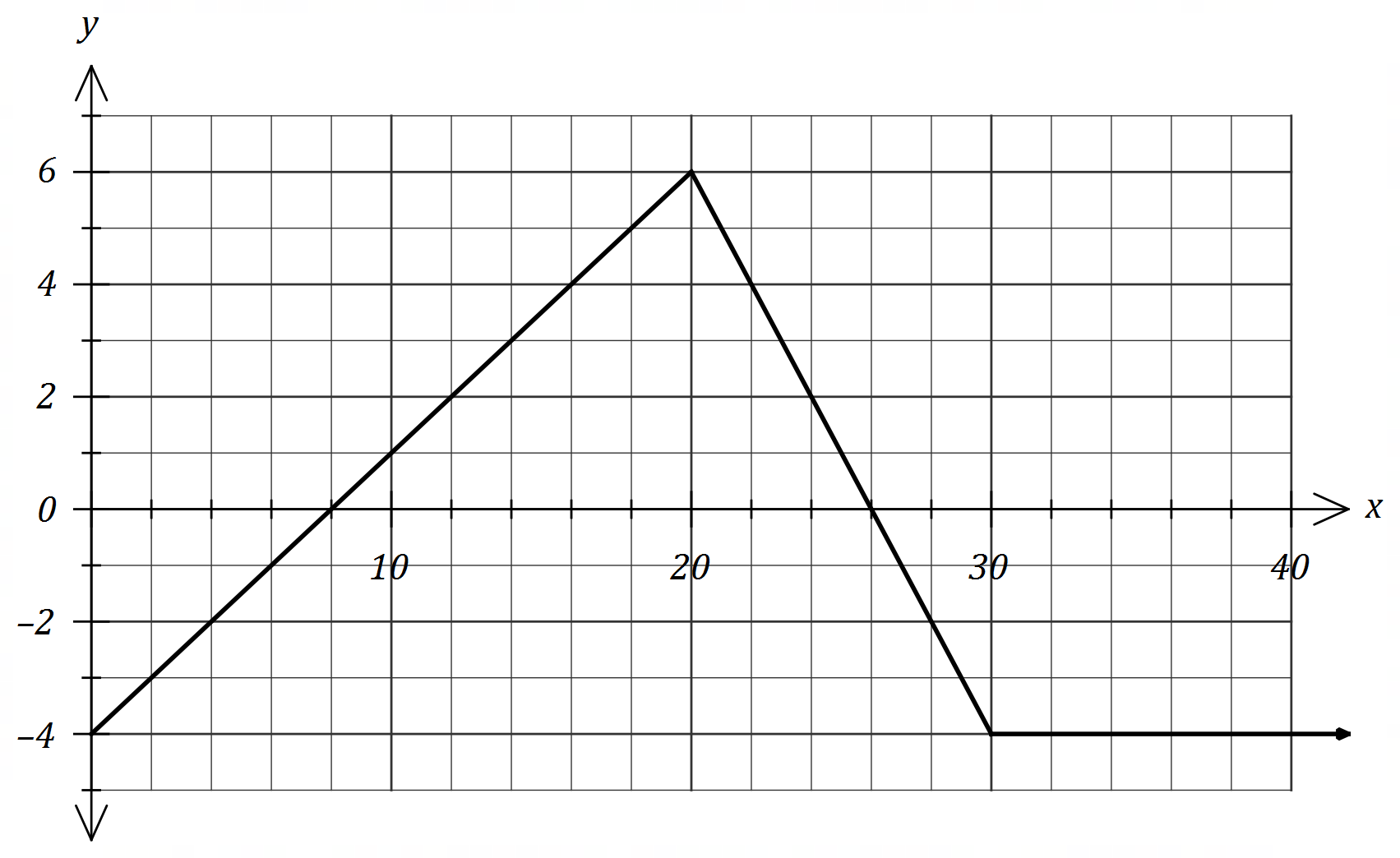
(d) Briefly describe how the closeness of the normal approximation would change if

(i) the sample size was larger. (1 mark)

(ii) the percentage of animals that are carriers was higher. (1 mark)

Question 16 (8 marks)

The graph of is shown below.



(a) Determine . (2 marks)

Let .

(b) Determine

(i) . (1 mark)

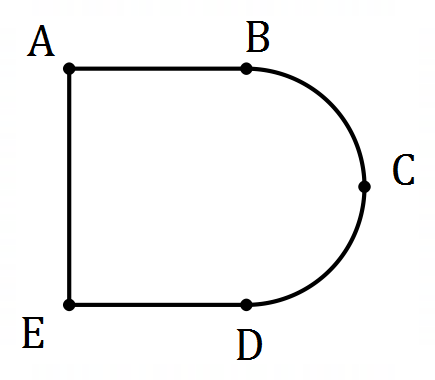
(ii) . (1 mark)

(c) Determine the coordinates of the maximum of the graph of . (2 marks)

(d) Determine the root of the graph of for . (2 marks)

Question 17 (7 marks)

When seen from above, an evaporation tank of area m2 has the shape of rectangle and semicircle of radius .



(a) If length , express in terms of and hence show that the perimeter, m, of the tank is given by (3 marks)

(b) Use a calculus method to determine the minimum perimeter of the tank. (4 marks)

Question 18 (7 marks)

A citrus farm grows Eureka lemons. Their weights are normally distributed with a mean of g and a standard deviation of g.

(a) Determine the probability that

(i) a randomly chosen lemon has a weight that exceeds g. (1 mark)

(ii) in a random sample of lemons, exactly have a weight that exceeds g.

(2 marks)

The farm classifies their lemons by size, so that the ratio of the number of small to medium to large lemons is .

(b) Determine the upper and lower bounds for the weight of a medium sized lemon. (2 marks)

(c) Determine the probability that when lemons are picked at random, the first small lemon is chosen on the th pick. (2 marks)

Question 19 (9 marks)

A particle moves along the -axis with initial position m and velocity m/s.

The acceleration of the particle after seconds is given by m/s2.

Between and the particle undergoes a change in displacement of m.

(a) Determine the value of the constant . (4 marks)

(b) Determine

(i) the maximum velocity of the particle. (2 marks)

(ii) the distance of the particle from the origin after seconds. (3 marks)

Question 20 (9 marks)

Researchers in a large city wish to determine a confidence interval for , the proportion of citizens who had used the city library at least once during the previous year. The margin of error of the interval is to be no more than .

(a) If the researchers had no reliable estimate for , determine the sample size they should take, noting **all** assumptions made. (5 marks)

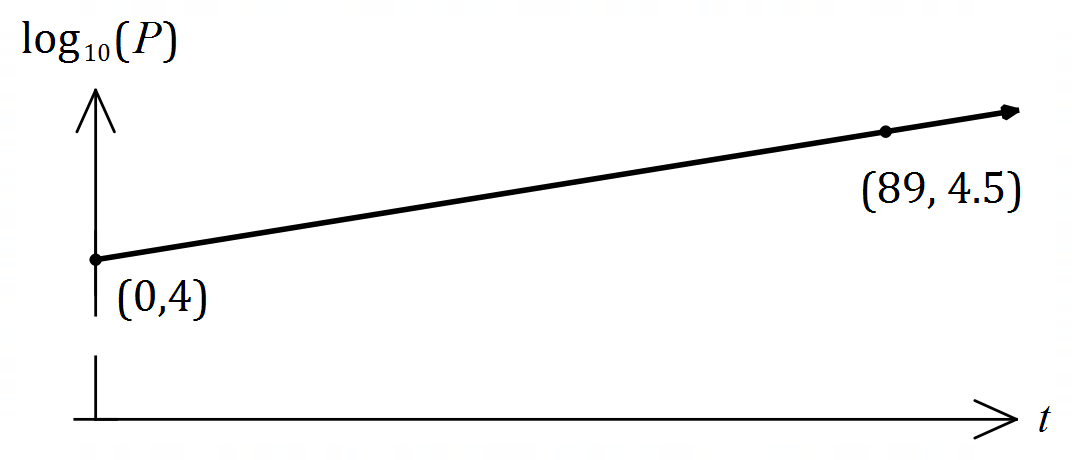
(b) The researchers were given access to data from a random sample of citizens collected a few years earlier. Of these, had used the city library at least once during the previous year.

(i) Determine the margin of error for a confidence interval for based on this sample. (2 marks)

(ii) The researchers used this data to decrease the sample size calculated in part (a). By how much did the sample size decrease? (2 marks)

Question 21 (9 marks)

The population of a species can be modelled by the equation , where and are constants and is the number of years since the population was first recorded. The graph below shows the linear relationship between and for the population over the past years and passes through the points and .



(a) Write an equation relating and . (2 marks)

(b) Determine the value of and the value of . (3 marks)

(c) Interpret the value of and the value of in the context of this model. (2 marks)

(d) Use the model to determine

(i) the population when . (1 mark)

(ii) the number of years for the population to reach . (1 mark)

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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